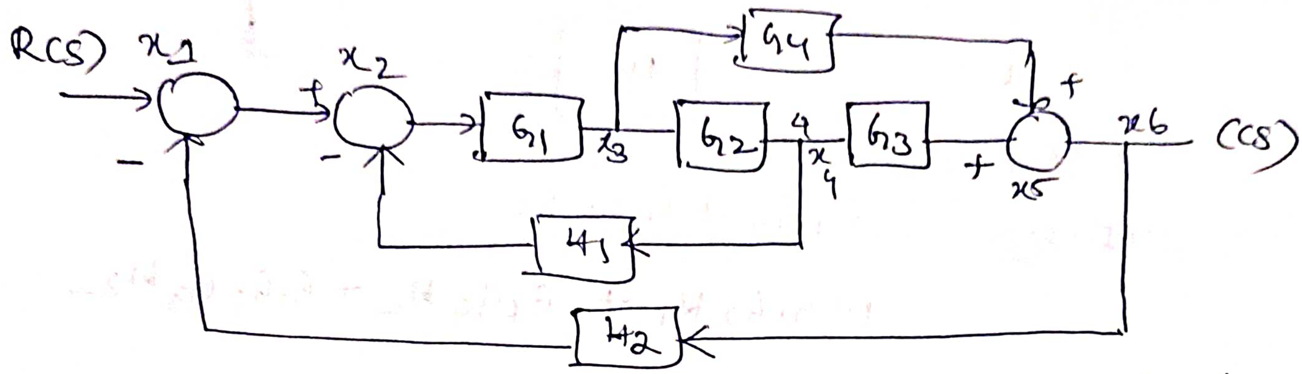
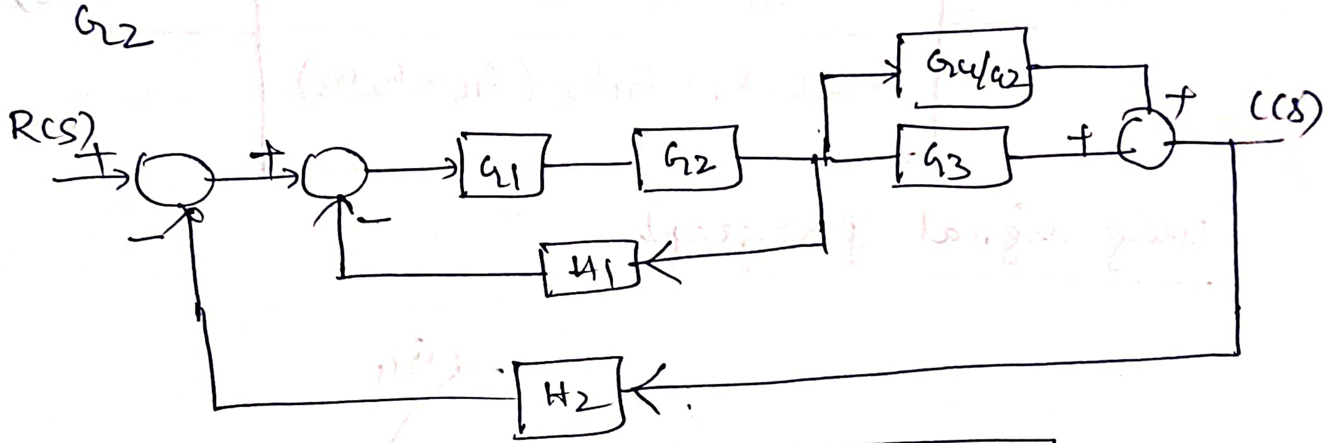
Module - 2 Problems on Block diagram Algebra & Signal flow graph

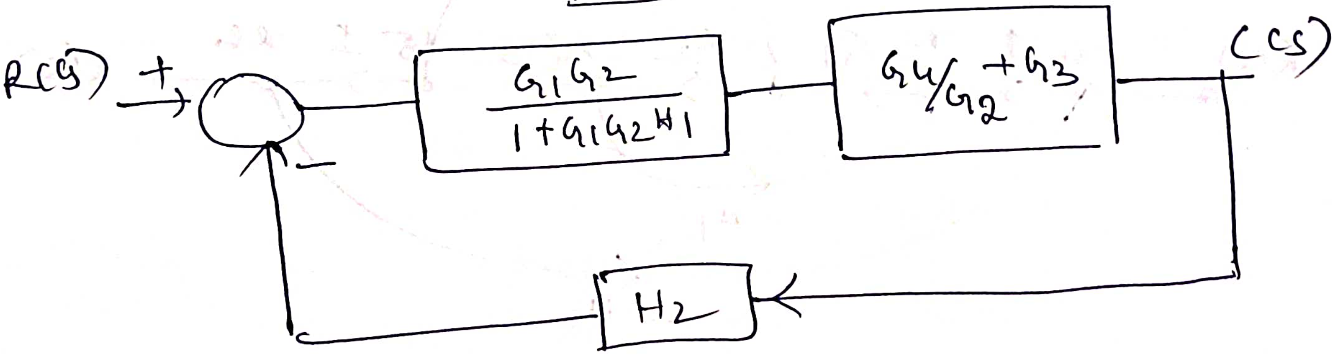
1) AVM

6) Obtain transfer function $\frac{C(s)}{R(s)}$ for the system shown in fig using Block diagram reduction technique & Mason's gain formula.

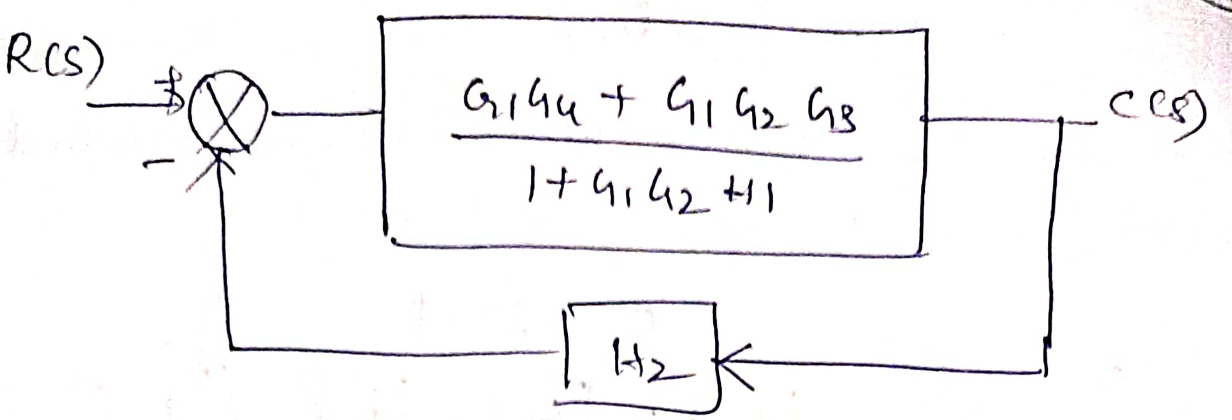


Step I Shift the take off point x_3 after block G_2

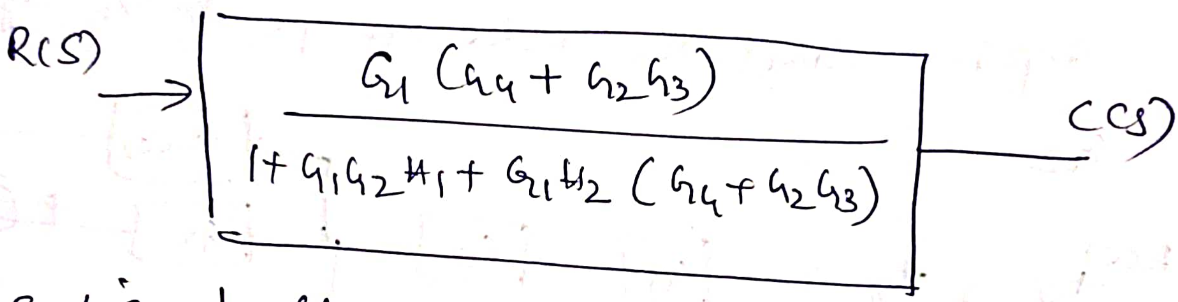




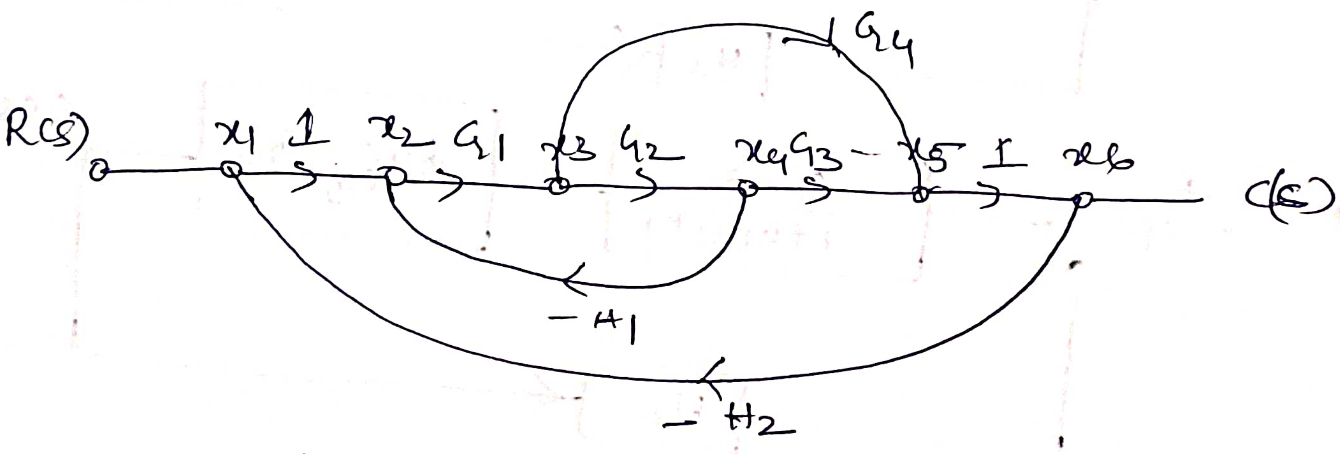
$$\frac{G_1\cancel{G_2}}{1+G_1G_2H_1} \times \frac{G_4+G_2G_3}{\cancel{G_2}} = \frac{G_1G_4+G_1G_2G_3}{1+G_1G_2H_1}$$



$$TF = \frac{G_1G_4 + G_1G_2G_3}{1+G_1G_2H_1 + G_1G_4H_2 + G_1G_2G_3H_2}$$



Using signal flowgraph



(1) Forward paths

P_1 with loop path gain $1 \times G_1G_2G_3 \times 1 = G_1G_2G_3$

P_2 $1 \times G_1 \times G_4 \times 1 = G_1G_4$

(2) Individual loops with loop gains

$L_1 = -G_1 G_2 H_1$

$L_2 = -G_1 G_4 H_2$

$L_3 = -G_1 G_2 G_3 H_2$

Note that all the three loops have common node x_1 to x_2

$\Delta = 1 -$ (sum of individual loop gains) + (sum of gain product of two non-touching loops) + ...

$= 1 - (L_1 + L_2 + L_3)$

$= 1 + G_1 G_2 H_1 + G_1 G_4 H_2 + G_1 G_2 G_3 H_2$

Mason's gain formula given by

$\frac{C(s)}{R(s)} = T = \frac{1}{\Delta} \sum P_k \Delta_k$ $\quad \Delta_1 = 1 \quad \Delta_2 = 1$

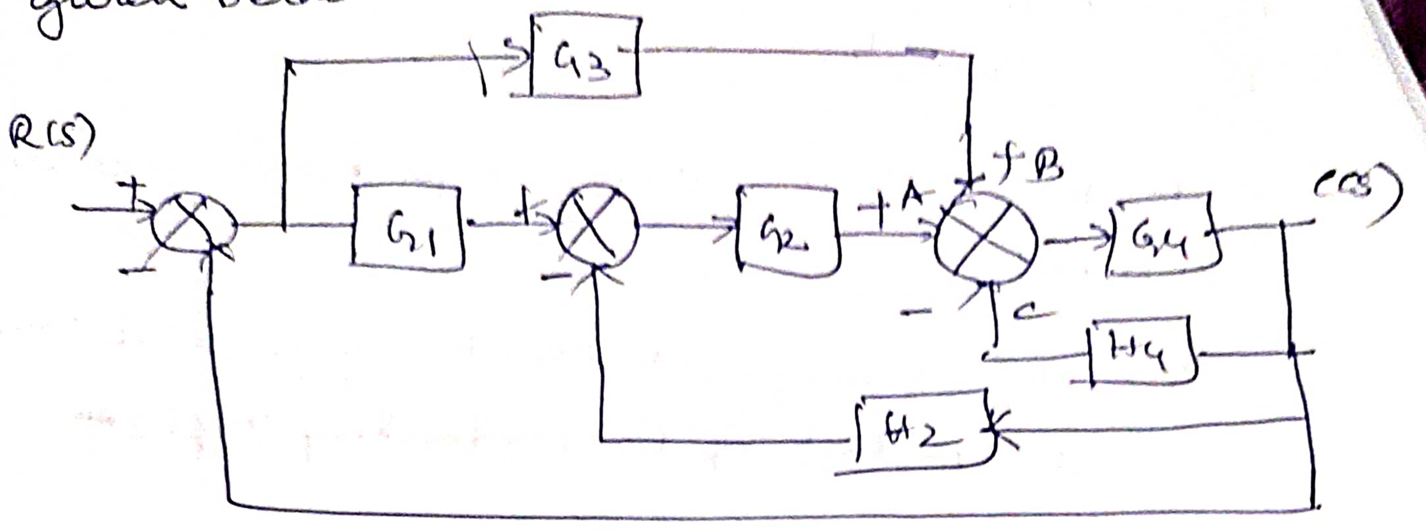
$$= \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{\cancel{G_1 G_4} + G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 H_1 + G_1 G_4 H_2 + G_1 G_2 G_3 H_2}$$

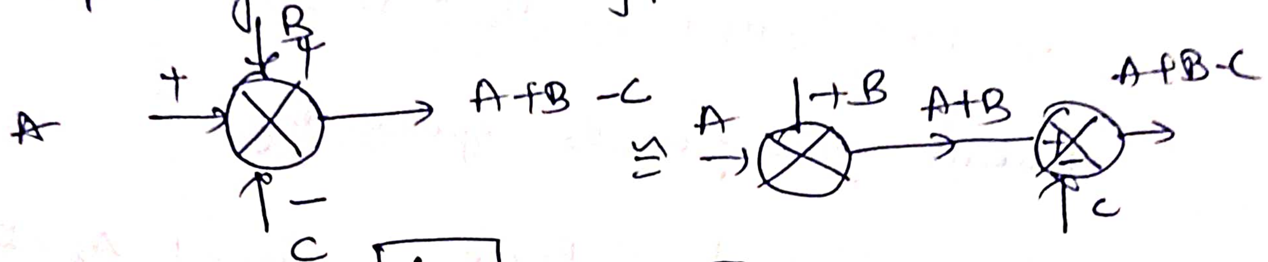
$$\boxed{T = \frac{G_1 (G_2 G_3 + G_4)}{1 + G_1 G_2 H_1 + G_1 H_2 (G_4 + G_2 G_3)}}$$

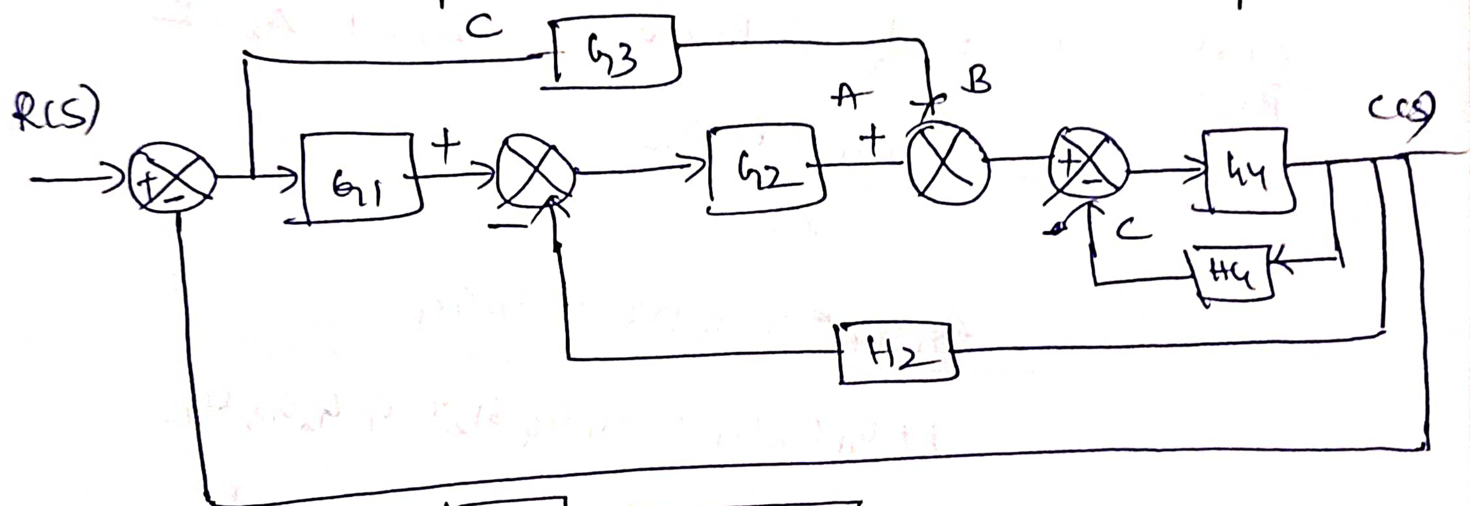
2a

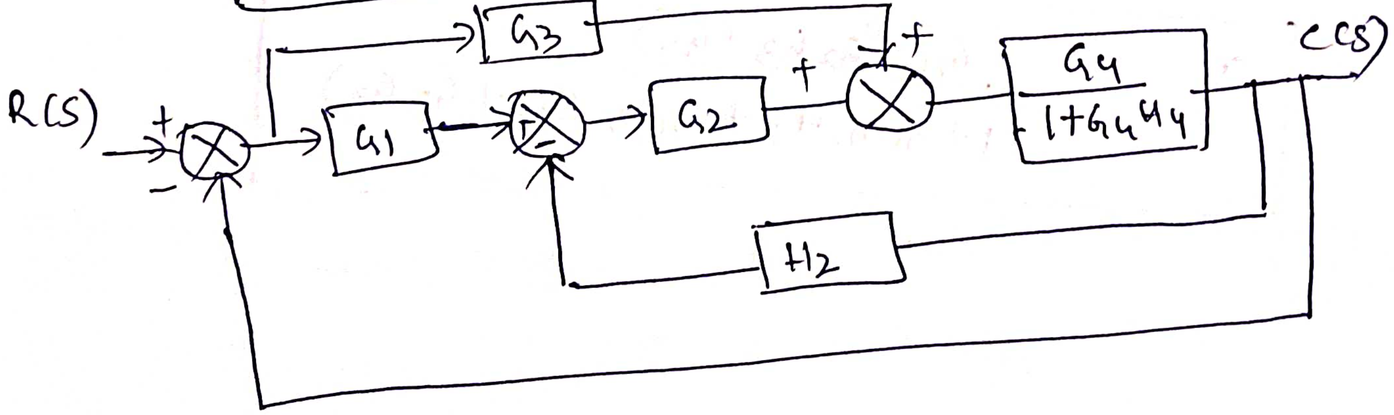
*) Find the overall transfer function for the system given below



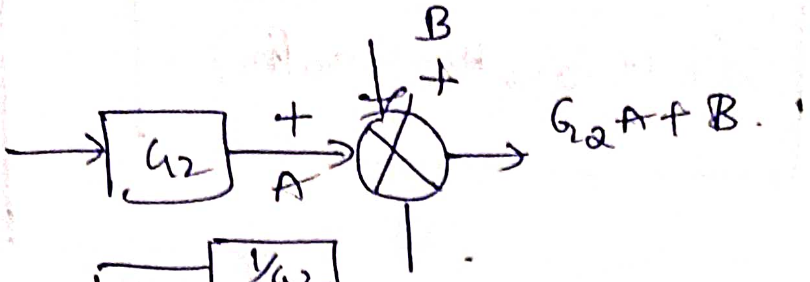
Splitting the summing point

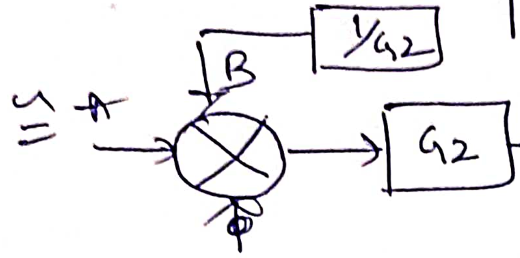






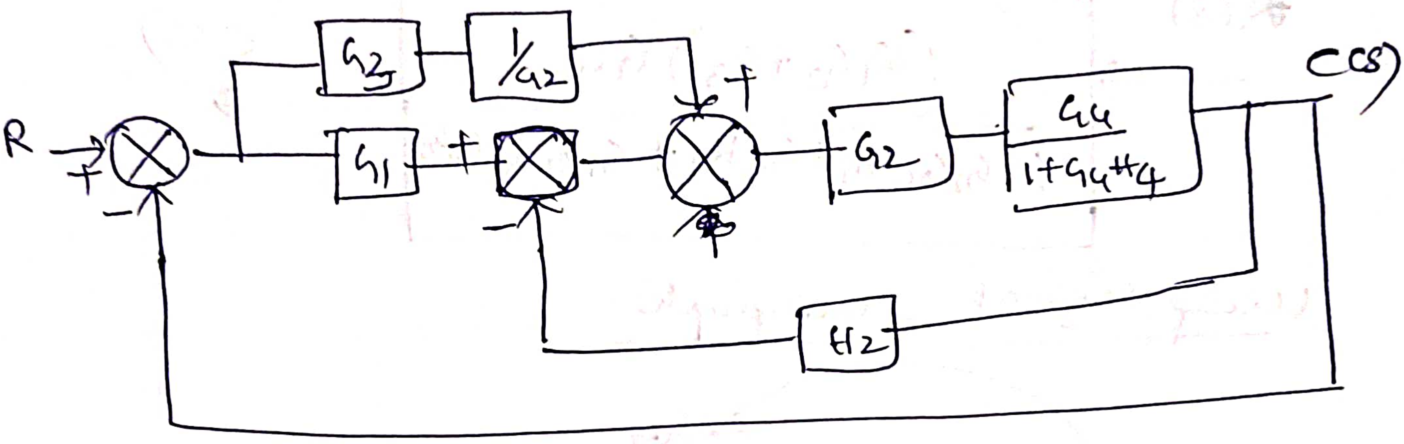
Shifting summing point before a block



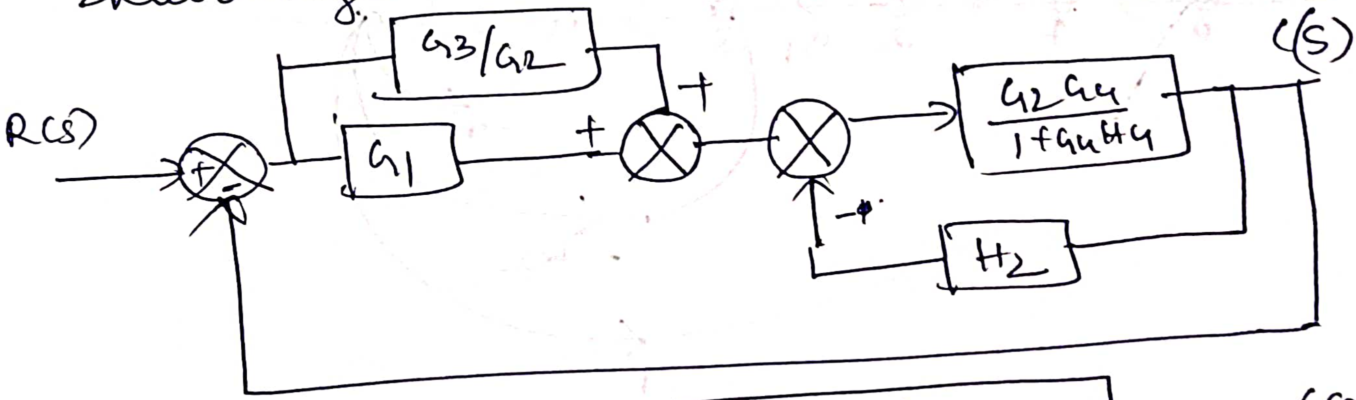


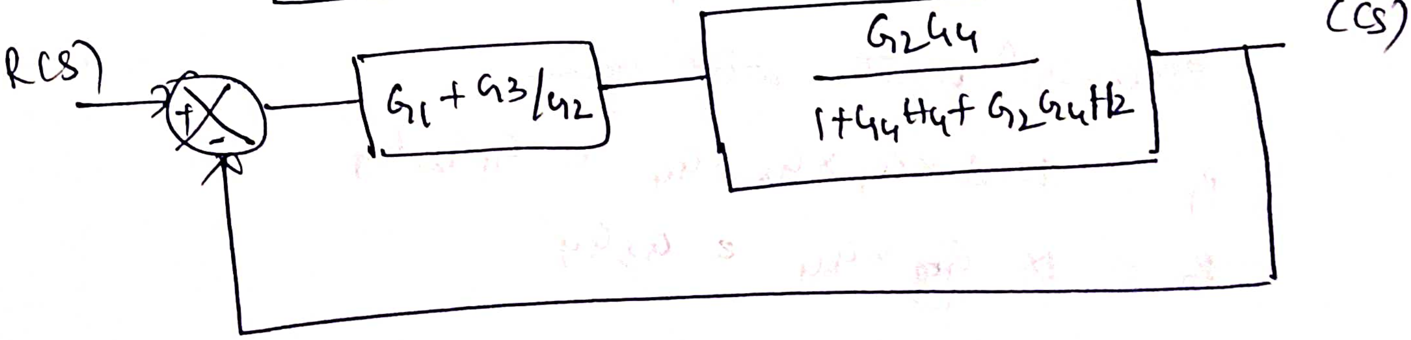
$$A G_2 + B/G_2 \times G_2$$

$$= A G_2 A + B.$$

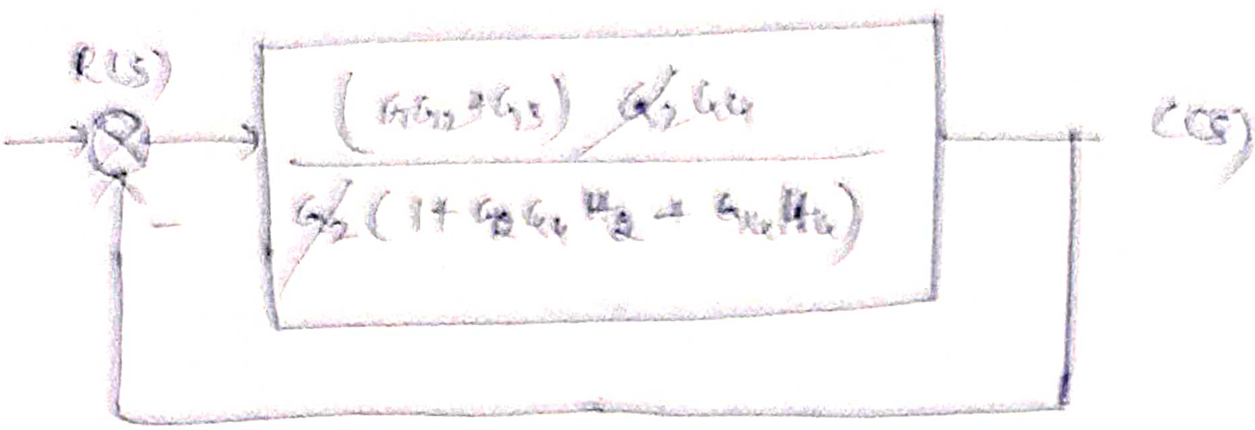


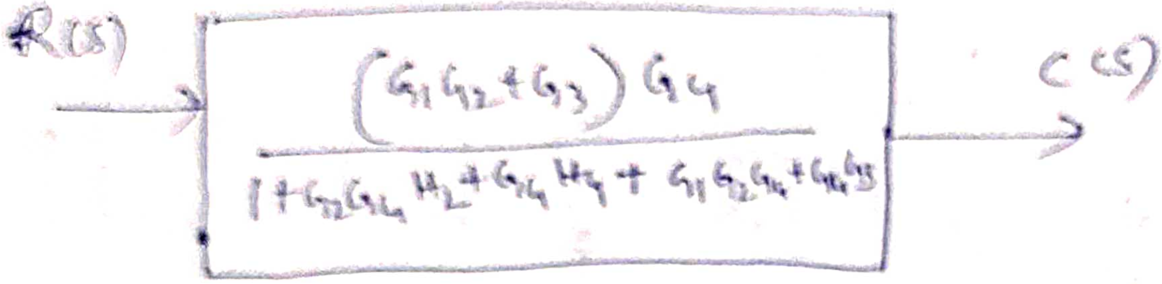
Interchange summations



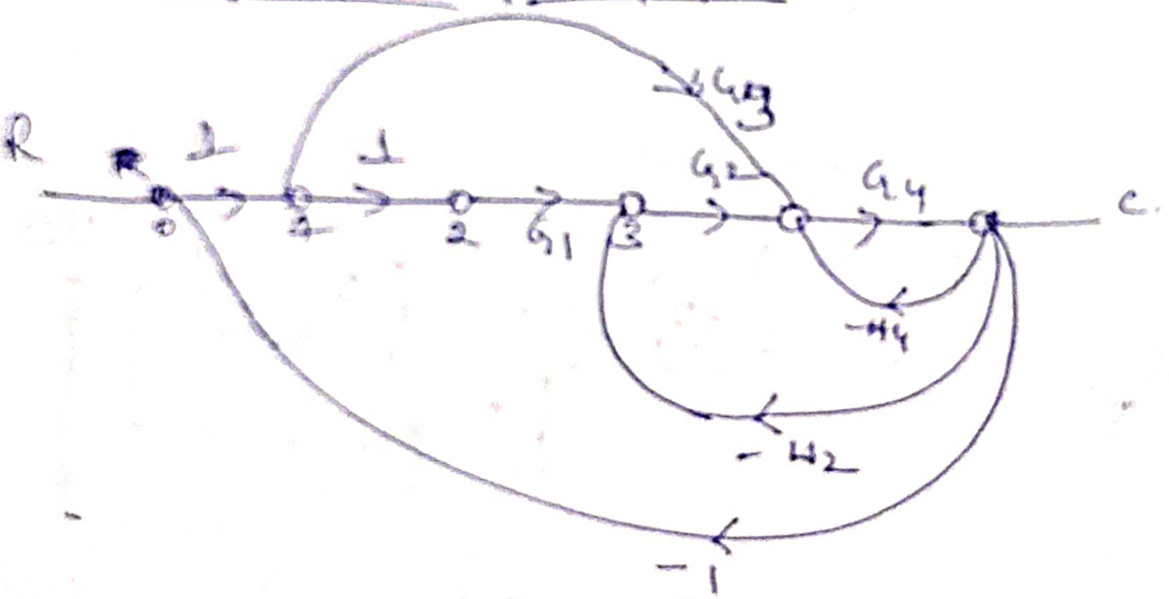


$H(S) = \Delta$





<u>Using signal flow graph</u>



Forward path with gain

$P_1 = 1 \times 1 \times G_1 \times G_2 \times G_4 = G_1G_2G_4$

$P_2 = 1 \times G_3 \times G_4 = G_3G_4$

loops with loopgains

$L_1 = -G_4 H_4$

$L_2 = -G_2 G_4 H_2$

$L_3 = -G_1 G_2 G_4$

$L_4 = G_3 G_4 \times -1 = -G_3 G_4$

All loops touch each other.

$\Delta_1 = \Delta_2 = 1$

$T = \frac{1}{\Delta} \sum_k P_k \Delta_k = \frac{1}{\Delta}(P_1 \Delta_1 + P_2 \Delta_2)$

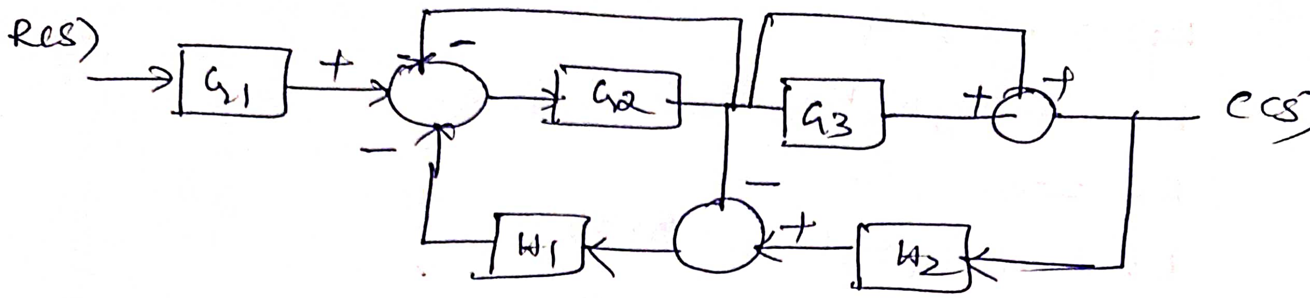
$\Delta = 1 - (L_1 + L_2 + L_3 + L_4)$

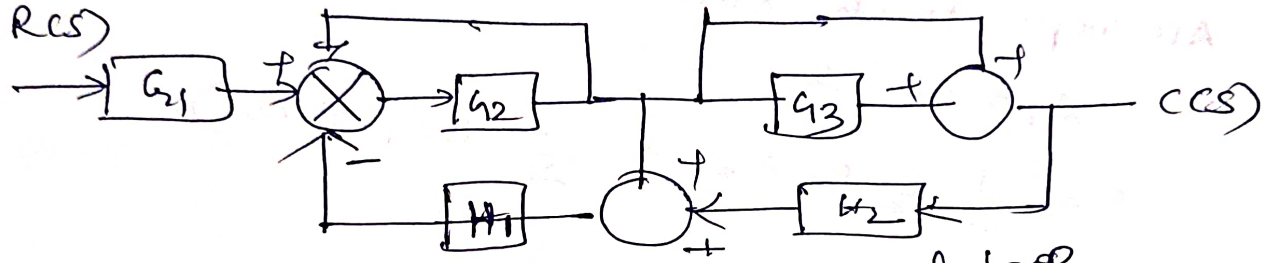
$= 1 + G_4 H_4 + G_2 G_4 H_2 + G_1 G_2 G_4 + G_3 G_4$

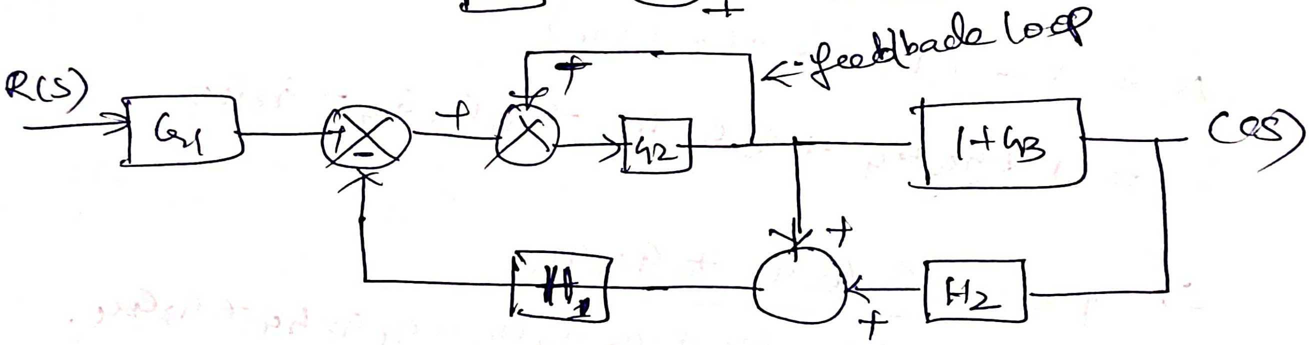
$$\therefore T = \frac{G_1 G_2 G_4 + G_3 G_4}{1 + G_4 H_4 + G_2 G_4 H_2 + G_1 G_2 G_4 + G_3 G_4}.$$

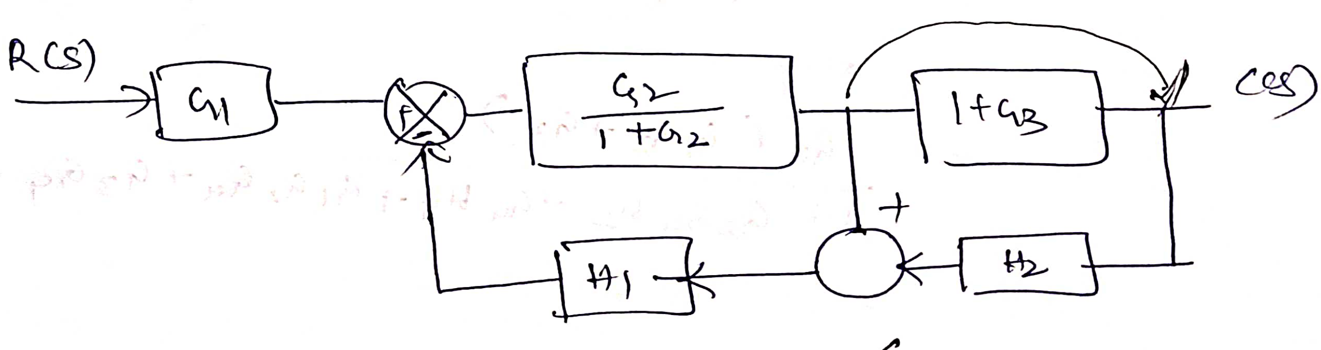
$$= \frac{G_4 (G_1 G_2 + G_3)}{1 + G_2 G_4 H_2 + G_4 H_4 + G_1 G_2 G_4 + G_3 G_4}$$

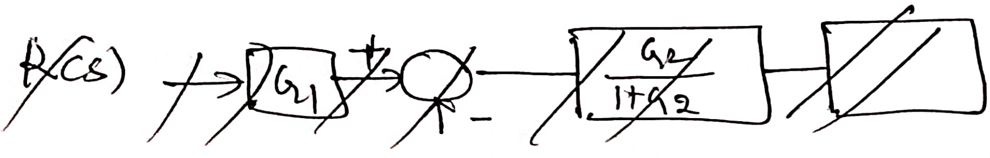
(8) Find over all transfer fun using Block diagram reduction technique or signal flow graph method











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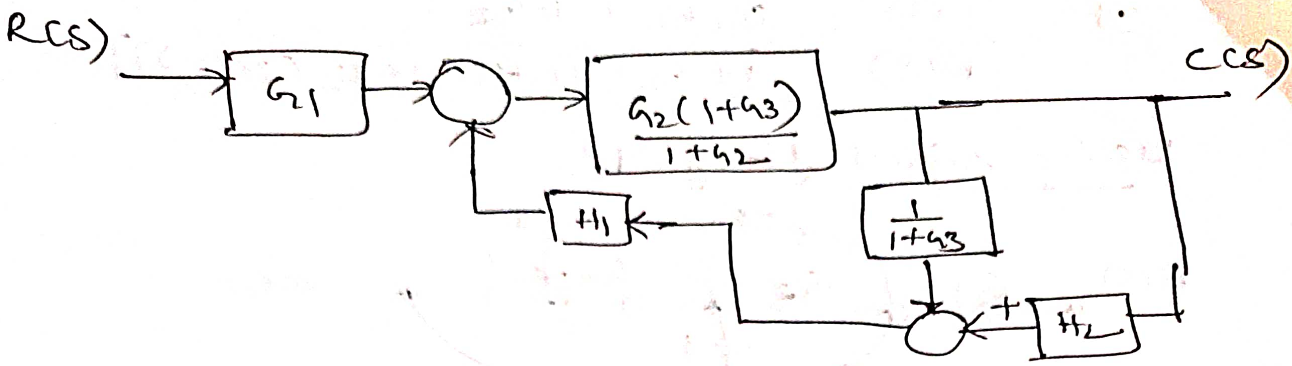
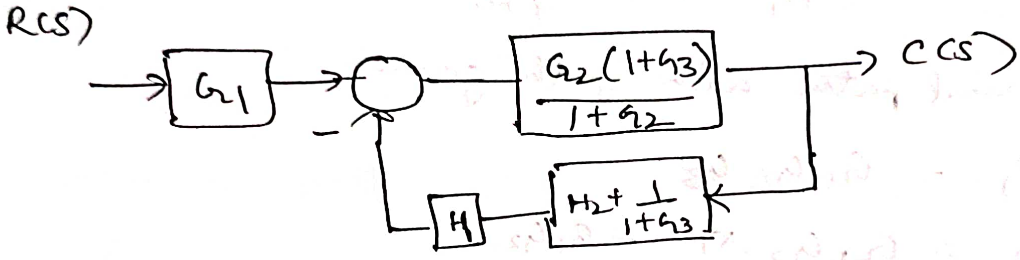
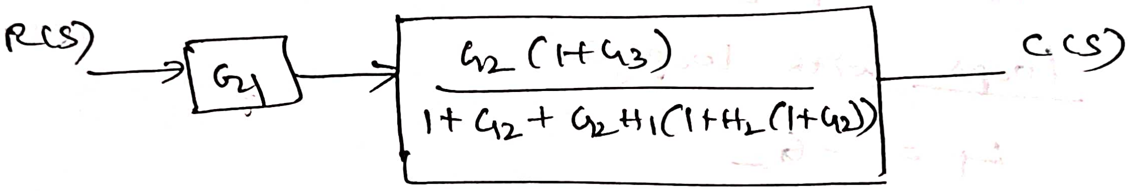






$$H(S) = H_1\left[\frac{H_2(1+G_3)+1}{1+G_3}\right]$$

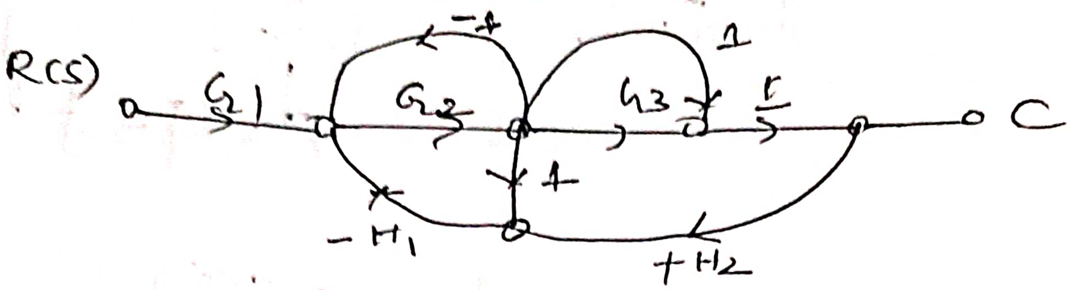
$$= \frac{H_1H_2 + H_1G_3 + H_1}{1+G_3}$$

$$TF = \frac{\dfrac{G_2(1+G_3)}{1+G_2}}{1 + \dfrac{G_2\,\cancel{(1+G_3)}}{1+G_2} \cdot \dfrac{H_1H_2 + H_1G_3 + H_1}{\cancel{(1+G_3)}}}$$

$$= \frac{G_2(1+G_3)}{1+G_2 + \cancel{G_2 H_1 [H_2} \cdot G_2H_1H_2 + G_2H_1G_3 + G_2H_1}$$

$$TF = \frac{C(s)}{R(s)} = \frac{G_1 G_2 (1+G_3)}{1+G_2 [1+H_1 [1+H_2 (1+G_3)]]}$$

Using signal flowgraph



① Forward path with path gain

$P_1 = G_1 G_2 G_3$

$P_2 = G_1 G_2 \times 1 = G_1 G_2$

Loops with loop gain

$L_1 = -G_2$

$L_2 = -G_2 H_1$

$L_3 = -G_2 G_3 H_1 H_2$

$L_4 = -G_2 H_1 H_2$

All loops touch each other ∴ No two nontouching loops

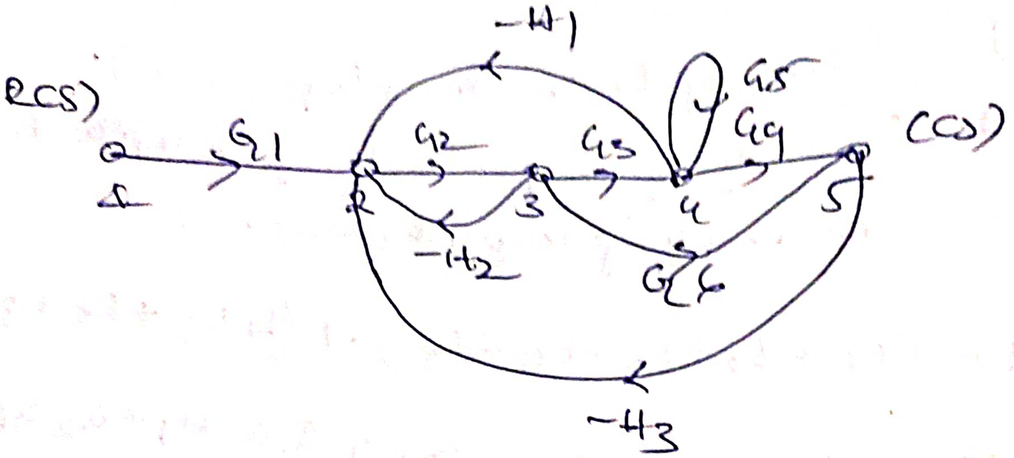
$$\Delta = 1 + G_2 + G_2 H_1 + G_2 G_3 H_1 H_2 + G_2 H_1 H_2$$
$$= 1 + G_2 (1 + H_1 + G_3 H_1 H_2 + \cancel{H_1} H_2)$$

$\Delta_1 = 1 \quad \Delta_2 = 1$ $\Rightarrow$ $1+G_2(1+H_1[1+H_2(G_3+1)])$

$$\therefore T = \frac{G_1 G_2 G_3 + G_1 G_2}{1+G_2 + G_2 H_1 + G_2 G_3 H_1 H_2 + G_2 H_1 H_2}$$

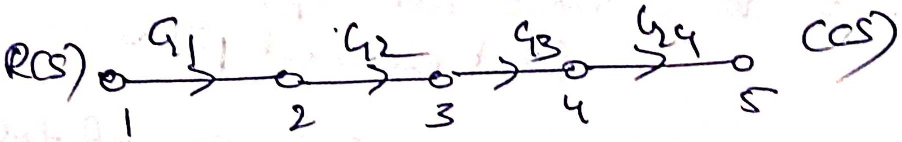
$$= \frac{\cancel{G_1 G_2 (G_1 G_3 + G_1)}}{\cancel{1+G_2(1+H_1) + H_1(G_3 H_2 + H_2)}} \cdot \frac{(1+G_3) G_1 G_2}{\Delta}$$

1) Find the overall gain C(s)/R(s) for the signal flowgraph shown in fig given below.



Forward path with path gains

$P_1 = G_1 G_2 G_3 G_4$



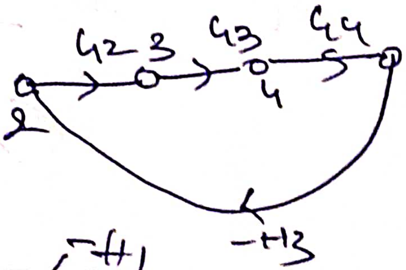
$P_2 = G_1 G_2 G_6$

Individual loop with loop gain

(1) $L_1 = -G_2 H_2$

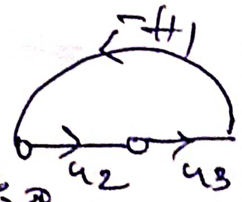
$L_2 = G_2 G_3 G_4 H_3$

$= -G_2 G_3 G_4 H_3$

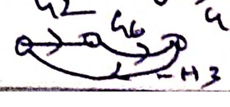


$L_3 = G_5$

$L_4 = -G_2 G_3 H_1$



$L_5 = -G_2 G_6 H_3$



Gain products of two non touching loop

$L_1 L_3 = -G_2 G_5 H_2$

$L_5 L_3 = -G_2 G_6 H_3 G_5$

$\Delta = 1 -$ (sum of Individual loop gains)
+ (sum of gain product of possible two nontouching loop with gain) - - - -

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_1 L_3 + L_5 L_3)$

$= 1 + G_2 H_2 + G_2 G_3 G_4 H_3 - G_5 + G_2 G_3 H_1 + G_2 G_6 H_3$
$- G_2 G_5 H_2 - G_2 G_6 H_3 G_5$

$\Delta_1 = 1$ (since all loops are touching first forward path

$\Delta_2 = 1 - G_5$ (not touching second forward path

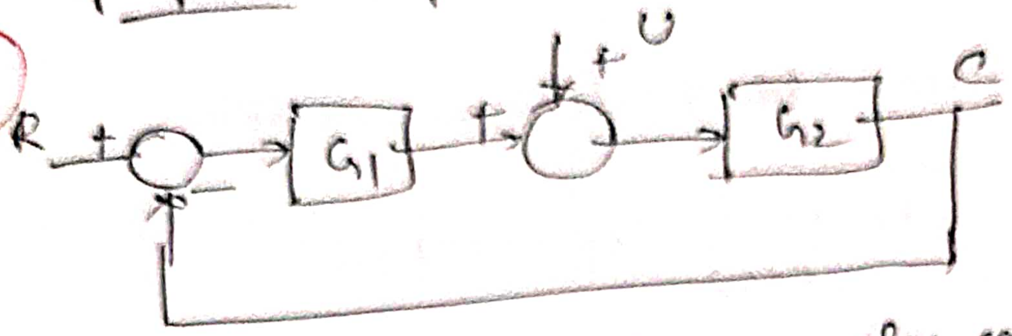
$\therefore \quad T = \frac{1}{\Delta} \sum_k P_k \Delta_k$

$= \frac{1}{\Delta} (P_1 \Delta_1 + P_2 \Delta_2)$

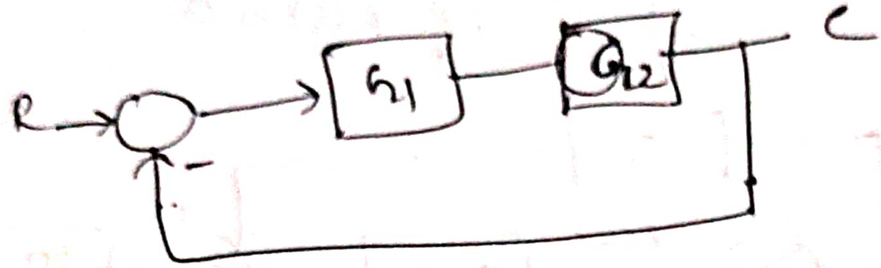
$$T = \frac{G_1 G_2 G_3 G_4 + G_1 G_2 G_6 (1 - G_5)}{1 + G_2 H_2 + G_2 G_3 G_4 H_3 - G_5 + G_2 G_3 H_1 + G_2 G_6 H_3 - G_2 G_5 H_2 - G_2 G_6 H_3 G_5}$$

Superposition of multiple inputs

(c) 10

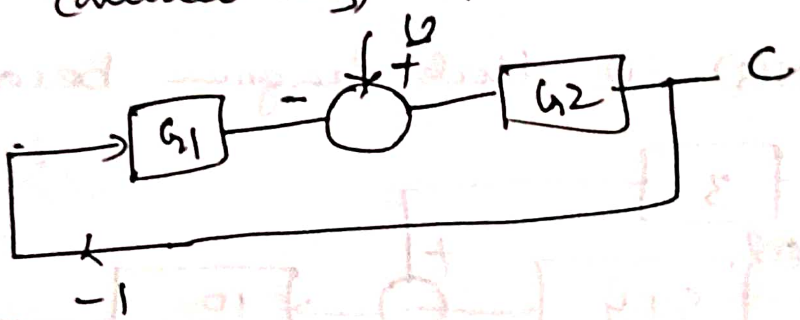


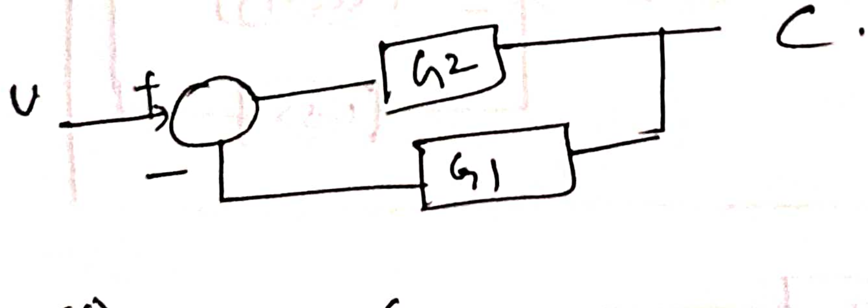
There are two inputs. Consider one at a time. Let $u = 0$



$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_1 G_2}$$

Next consider $u(s)$ put $R(s) = 0$





$$\frac{C(s)}{U(s)} = \frac{G_2}{1 + G_1 G_2}$$

∴ output due to u

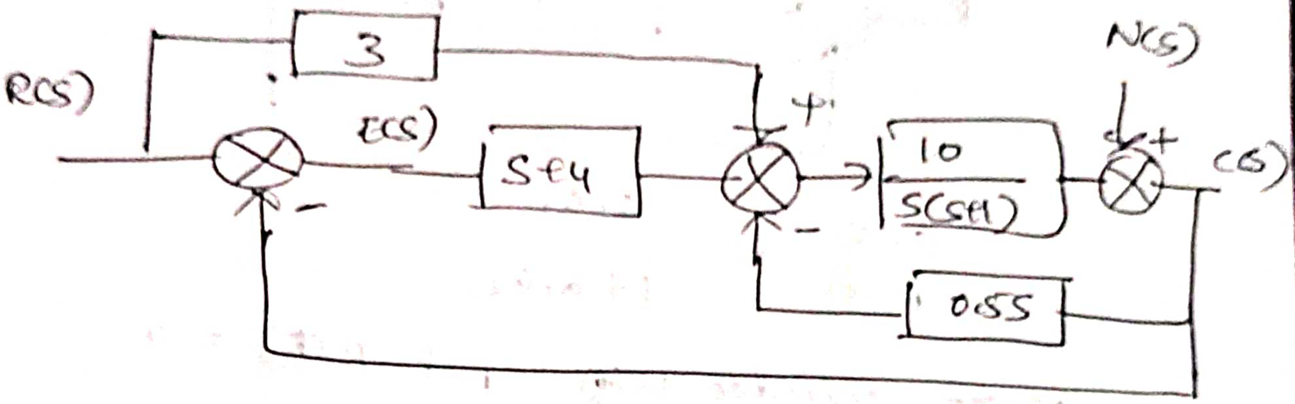
$$C(s) = \frac{G_2}{1+G_1 G_2} U(s)$$

The system block diagram is shown find

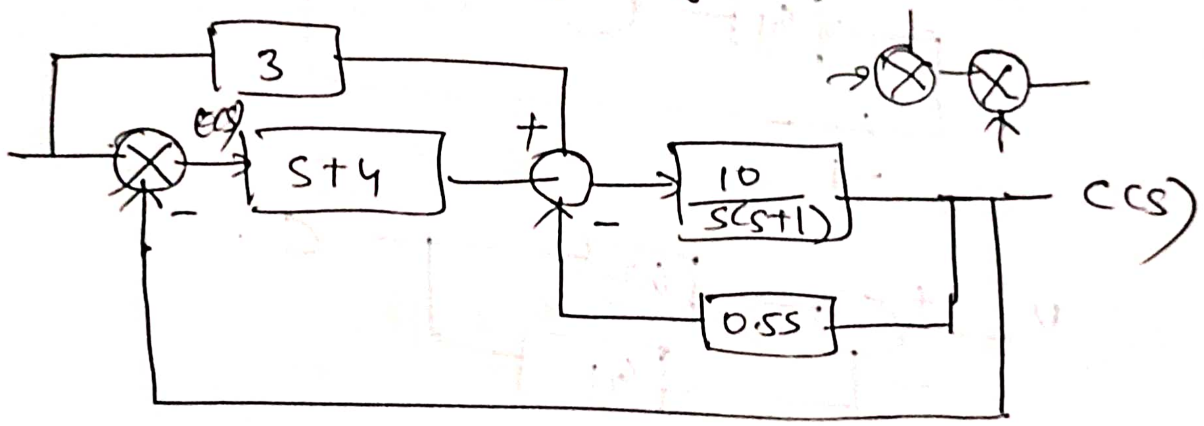
i) $\frac{C(s)}{E(s)}$ if $N(s) = 0$

ii) $\frac{C(s)}{R(s)}$ if ~~N(s)=0~~ . $N(s) = 0$

iii) $\frac{C(s)}{N(s)}$ if $R(s) = 0$



i) with $N(s) = 0$ block diagram becomes



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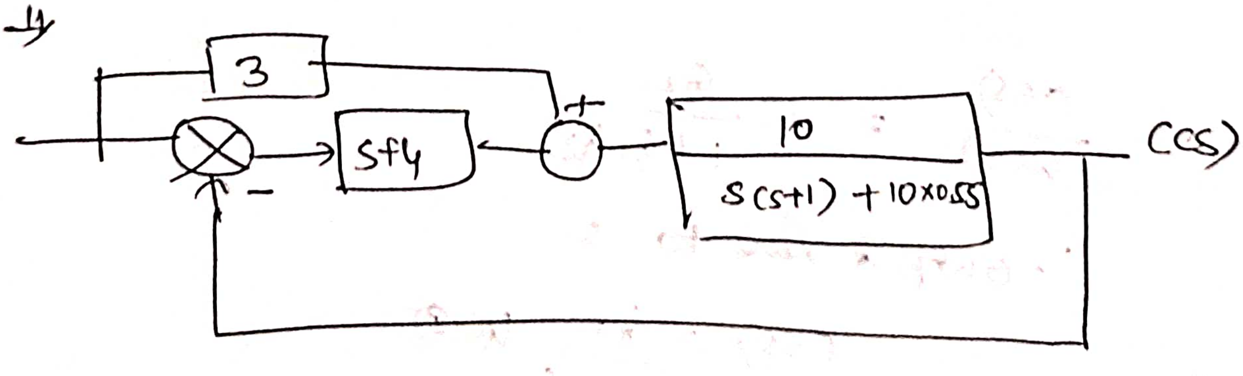
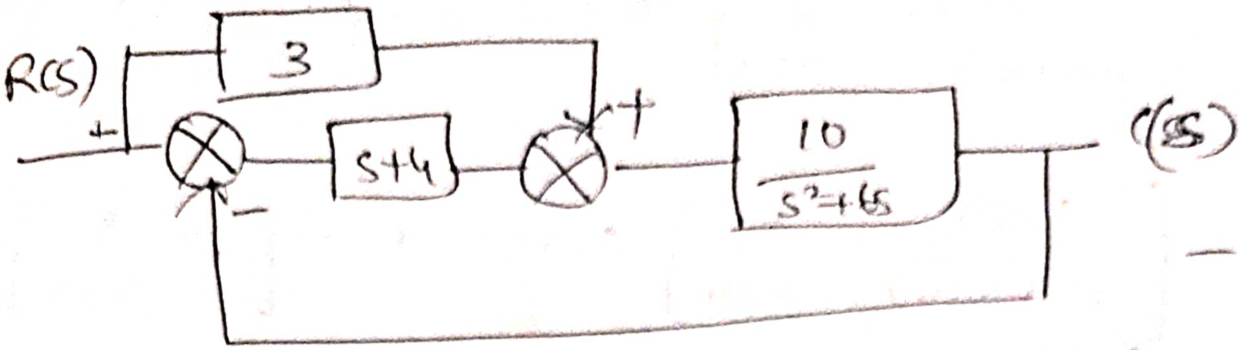
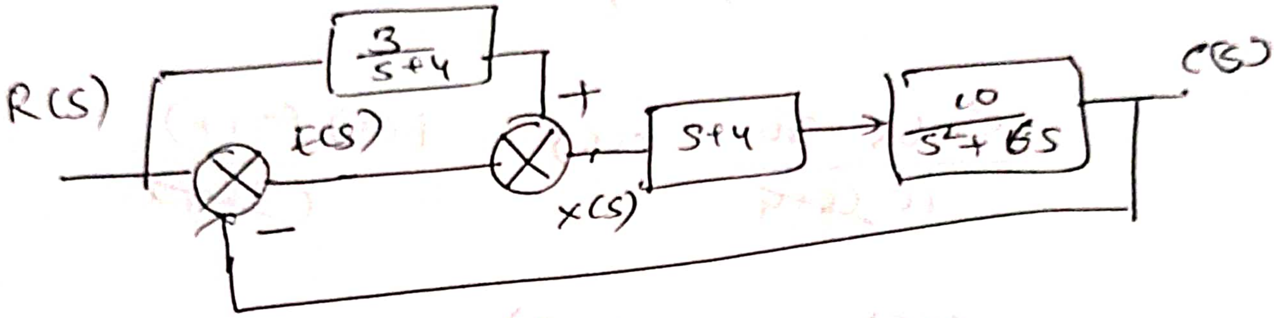




Shift summing point before block



$$E(S) = R(S) - C(S) \quad — (1)$$

$$C(S) = X(S)\,\frac{10(S+4)}{S^2+6S} \Rightarrow X(S) = \frac{S^2+6S}{10(S+4)}\,C(S) \quad — (2)$$

$$E(S) = R(S) - X(S)\,\frac{10(S+4)}{S^2+6S} \quad — (3)$$

$$\Rightarrow R(S) = E(S) + X(S)\,\frac{10(S+4)}{S^2+6S}$$

$$X(S) = E(S) + \frac{3}{S+4}\,R(S)$$

~~E(S) =~~

$$\frac{S^2+6S}{10(S+4)}\,C(S) = E(S) + \left[E(S) + \left(\frac{\cancel{S^2+6S}}{10\cancel{(S+4)}}\,C(S)\,.\,\frac{10\cancel{(S+4)}}{\cancel{S^2+6S}}\right) \times \frac{3}{S+4}\right)$$

$$\frac{S^2+6S}{10(S+4)}\,C(S) = E(S) + \frac{E(S)\,3}{S+4} + \frac{C(S)\,.\,3}{S+4}$$

$$\left(\frac{s^2+6s}{10s+40} - \frac{s}{s+4}\right) C(s) = E(s)\left[1+\frac{3}{s+4}\right]$$

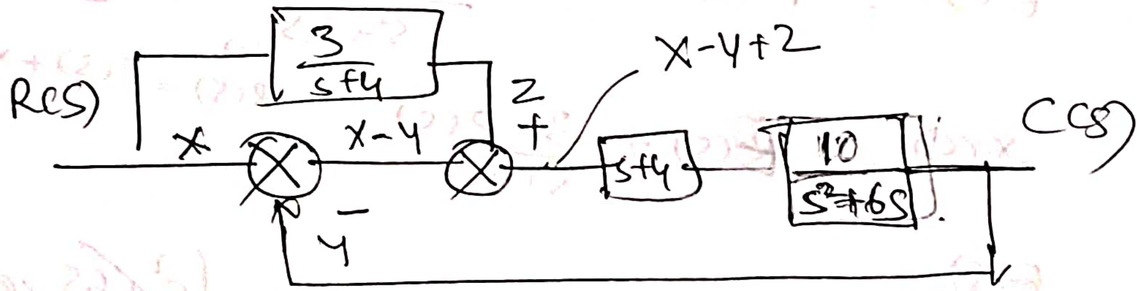
$$\left[\frac{s^2+6s}{10(s+4)} - \frac{s}{s+4}\right] C(s) = E(s)\left[1+\frac{3}{s+4}\right]$$

$$\left[\frac{s^2+6s-30}{10(s+4)}\right] C(s) = E(s)\left[\frac{s+4+3}{s+4}\right]$$

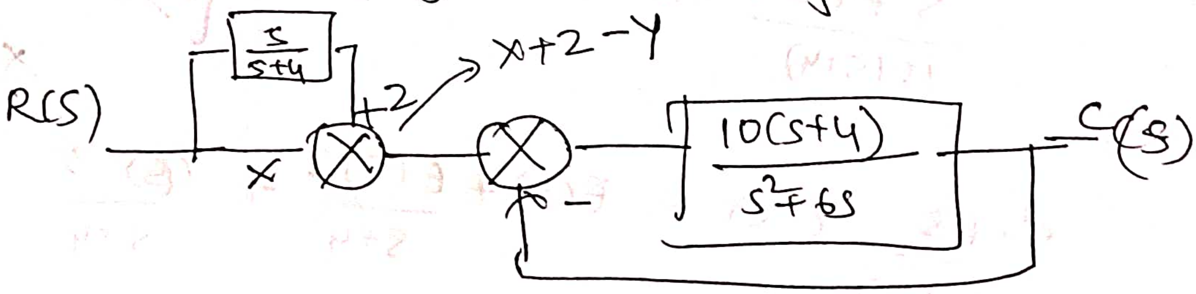
$$\frac{s^2+6s-30}{10\cancel{(s+4)}} C(s) = E(s)\frac{(s+7)}{\cancel{(s+4)}}$$

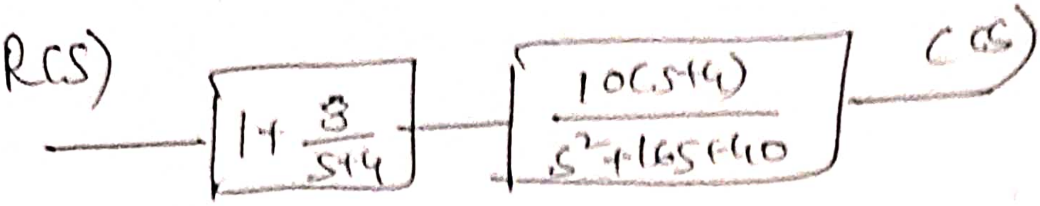
$$\frac{C(s)}{E(s)} = \frac{10(s+7)}{s^2+6s-30}. \quad \text{when } N(s)=0$$

(ii) $\frac{C(s)}{R(s)}$ & $N(s) = 0$.



Interchanging the summing points

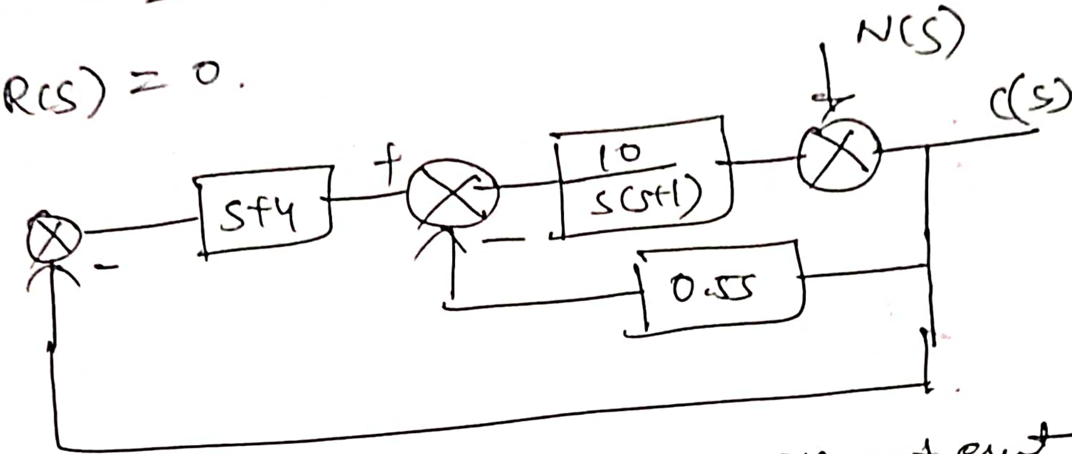




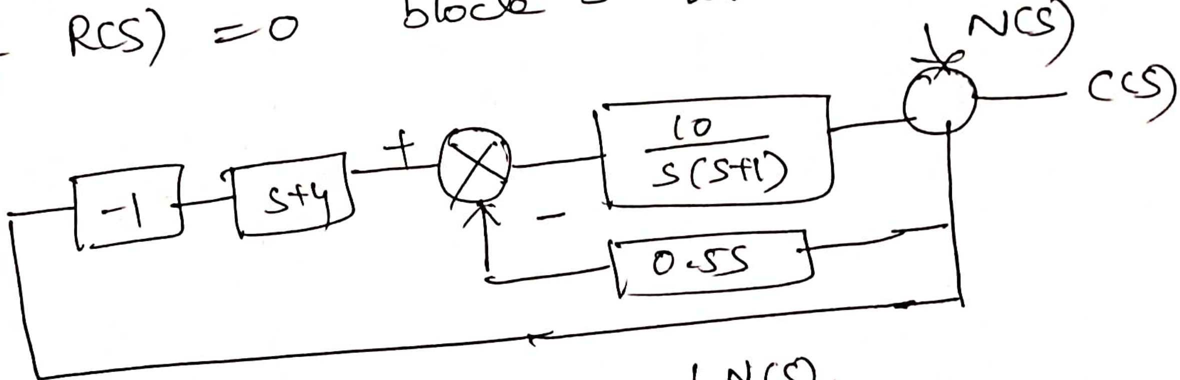
$$\boxed{\frac{C(s)}{R(s)} = \frac{s+4+3}{\cancel{s+4}} \times \frac{10\,\cancel{(s+4)}}{s^2+16s+40}}$$

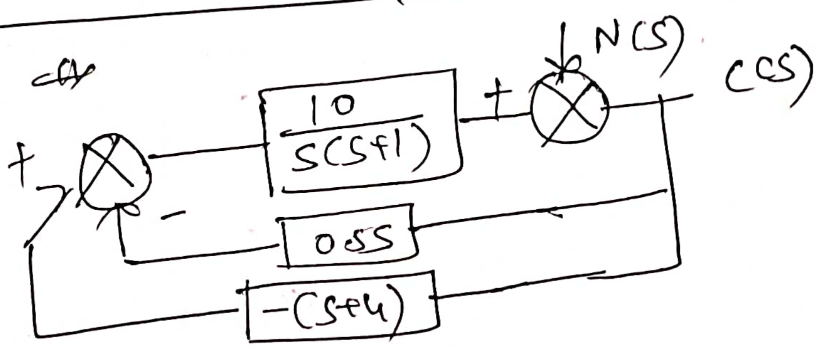
$$\frac{C(s)}{R(s)} = \frac{10(s+7)}{s^2+16s+40}$$

(iii) $R(s) = 0.$

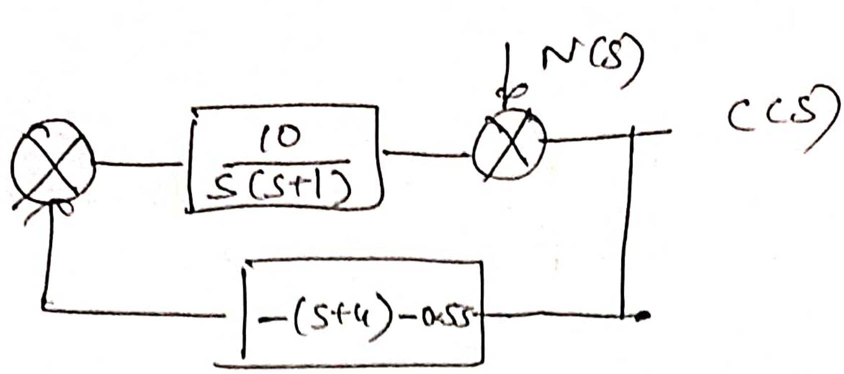


If $R(s) = 0$ block '3' will not exist

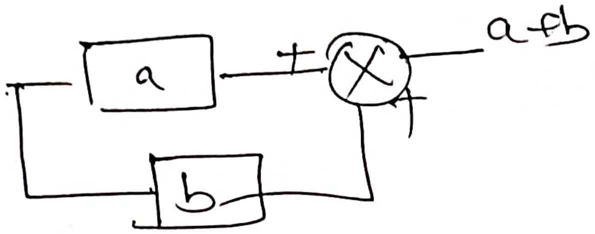


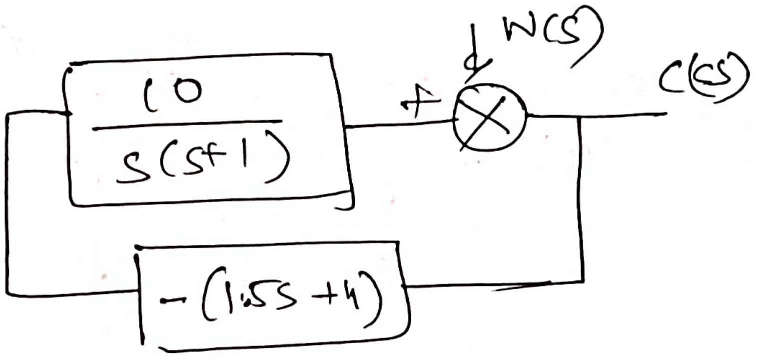


7a

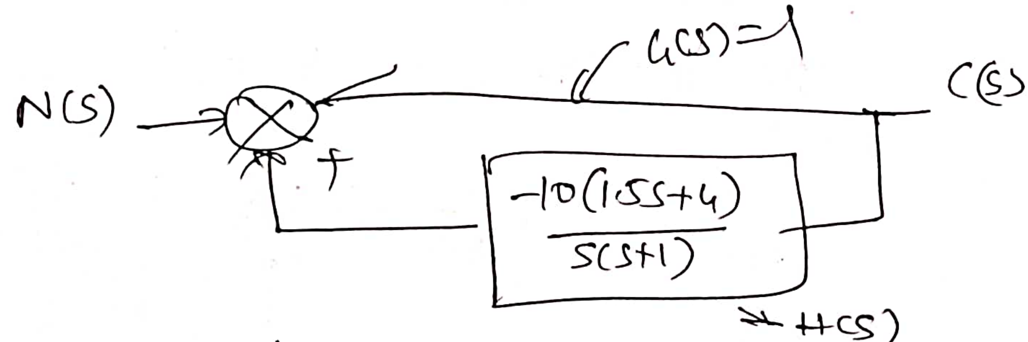


0.55
s -1(s+4) } two blocks in parallel





$\Rightarrow$



$$\frac{C(s)}{N(s)} = \frac{1}{1 - \left[\frac{-10(1.55s+4)}{s(s+1)}\right]}$$

$$= \frac{s(s+1)}{s^2+16s+40}$$